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RELATIVE MASSES OF BINARY STARS

BY

STEPHEN MARSHALL HADLEY

A THESIS SUBMITTED FOR THE DEGREE OF DOCTOR OF PHILOSOPHY,
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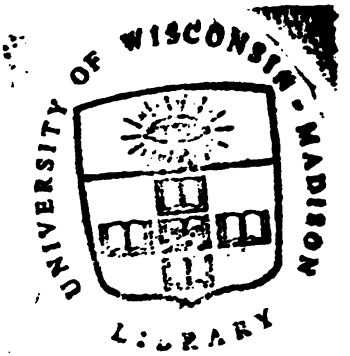

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RELATIVE MASSES OF BINARY STARS.

STEPHEN MARSHALL HADLEY.

I.

On the Motion of the Center of Gravity of a Binary Star With Respect to a Neighboring Star, and the Relative Masses of the Two Components of the Binary.

Let A and B denote the two components of the binary of which A is the brighter, and let C represent the neighboring star.

If we have a series of observations of A and B extending over an entire period of a binary or over the greater part of a period, giving the micrometrical measures of the position angles and distances of B with respect to A, also a series of observations of A and C, giving the micrometrical measurements of the position angles and distances of C with respect to A, the relative masses of A and B and the motion of their center of gravity with respect to C may be found from these observations.

In the case of a close binary if the position angles, θ^1 and distances ρ^1, of C with respect to $\frac{AB}{2}$ are given, the coördinates (θ, ρ) of C with respect to A may be found as follows:

Let G be the middle point of the line joining A and B.

(p, s) the position angle and distance of B with respect to A.

ϕ the angle ACG.

From the triangle ACG. the following equations are written:

$$\sin\rho\,\sin\phi = \sin\frac{s}{2}\sin(\theta' - p)$$

$$\sin\rho\cos\phi = \sin\rho'\cos\frac{s}{2} + \cos\rho'\sin\frac{s}{2}\cos(\theta' - p)$$

$$\cos\rho = \cos\rho'\cos\frac{s}{2} - \sin\rho'\sin\frac{s}{2}\cos(\theta' - p).$$

Noting that ρ' and s are very small quantities their cubes may be neglected, and putting

$$\frac{s}{2}\sin(\theta' - p) = \lambda\,,\quad \frac{s}{2}\cos(\theta' - p) = \mu$$

we have, by developing cos ρ,

$$1 - \frac{\rho^2}{2} = \left(1 - \frac{\rho'^2}{2}\right)\left(1 - \frac{s^2}{8}\right) - \rho'\mu,$$

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whence

$$\rho^2 = \rho'^2 + 2\rho'\mu \text{ (neglecting higher terms)} \qquad (1)$$

$$\tan \phi = \frac{\lambda}{\rho' + \mu}$$

$$\theta = \theta' \pm \phi . \qquad (2)$$

Whether ϕ is to be added or subtracted must be determined from the relative values of p and θ.

Development of Formulæ.

Let

ρ = the angular distance from A to C,
θ = position angle of C with respect to A,
x, y = the rectangular coördinates of A with C as origin,
x', y' = the rectangular coördinates of the center of gravity O, of A and B, with C as origin,
ξ, η = the coördinates of O with A as origin,
s = the radius vector of AB,
p = the position angle of B with respect to A,
k = the ratio of the distance of the center of gravity of A and B from A to the whole distance between A and B.

The coördinates of the center of gravity, O, of A and B are

$$x' = x + \xi$$
$$y' = y + \eta$$

With the origin at A these reduce to

$$x' = -ks \cos p + \rho \cos \theta$$
$$y' = -ks \sin p + \rho \sin \theta$$

On condition that the motion of the center of gravity, O, of the binary with respect to C is rectilinear its coördinates will be of the form

$$x' = \alpha + \beta\tau$$
$$y' = \mu + \nu\tau$$

in which α is the difference in declination at some assumed epoch between the center of gravity of A and B and the third star C, β is the relative annual proper motion in declination between C and the center of gravity of A and B, when τ is expressed in years.

τ is the interval of time between the date of observation and the assumed epoch.

μ, ν and τ, are corresponding data for motion at right angles to the hour circle.

By equating the above values of x' and y' the following equations are obtained:

$$\left.\begin{aligned} \alpha + \beta\tau + ks \cos p &= \rho \cos \theta \\ \mu + \nu\tau + ks \sin p &= \rho \sin \theta \end{aligned}\right\} \qquad (3)$$

In these equations τ, s, p, ρ *and* θ will be known from the record of the observations. Each pair of observations will furnish two equations of the form of equations (3).

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The data of the problem will be the observed position angles, θ, and distances, ρ, of the optical companion C, referred to the bright component A, and similar measurements, p and s, of the component B of the binary, for the dates of the available observations of C.

For the application of these formulæ to the cases of the binary systems whose relative masses are hereinafter investigated I have made use of all the observed data contained in the library of Washburn Observatory and in the presentation of these data the following abbreviations will be used in the tables.

Σ = William Struve
0Σ = Otto Struve
HΣ = Herman Struve
Δ = Dembowski
W = Winlock
Du = Dunér
β = Burnham
Pe = Perotin
Gl = Glasenapp
S = Comas-Sola
C = Comstock
Sp = Schiaparelli
L = Lewis
Pr = Pritchett
Ba = Barnard
A = Aitkin
Hl = Hall

a = Annals H. C. O.
m = Astronomische Nachrichten,
q = Astronomical Journal,
c = Catalogue des Étoiles Doubles et Multiplies, Flammarion,
f = Double Stars, Burnham,
g = Étoiles Doubles, Dunér,
s = Measures Micrometriques d' Étoiles Doubles Le Professeur S. de' Glasenapp,
i = Misure di Stelle Doppie II, Dembowski,
j = Monthly Notices R. A. S.
*b*1 = Observations de Pulkova, Vol. IX,
*b*2 = Observations de Pulkova, Vol. X,
h = Observations of Double Stars of Asaph Hall,
d = Publications de L'Observatoire Central Nicolas, Series II, Vol. XII.
l = Publications of the Yerkes Observatory Vol. I.
n = Publicaziona Del Reale Osservatorio Di Brera in Milano No. XXXIII.
o = Unpublished Observations by Professor Comstock.

II.

Determination of the Relative Masses in the System σ Coronæ.

$\alpha = 16^h\ 11^m \qquad \delta = +34^\circ\ 7'$

This star was discovered by Sir William Herschel August 17th, 1780. The star has been well observed over a sufficient arc of its orbit so that the elements of the orbit are well determined. According to See* the period is 370.0 years. The component is approaching apastron which it will reach in the early part of the next century, so that during the next two centuries its motion

* Evolution of the Stellar Systems Vol. I.

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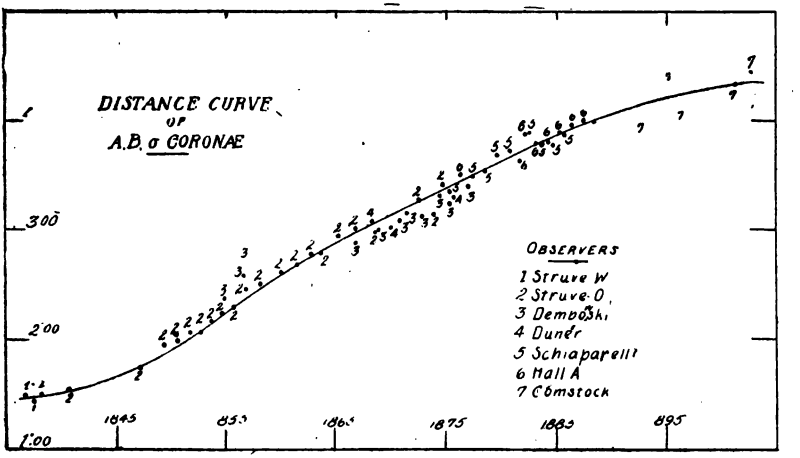
will be very slow. Consequently observations for many years to come will probably not add much to our present knowledge of this pair. In 1825 a third star *C* was observed by Sir J. South at a distance of about 40″ from the principal star; this distance has increased to more than 60″ but the difference is satisfactorily accounted for by the proper motion of the system. The magnitudes of the components *A* and *B* and of the third star C are about 4.5, 5.0 and 10.2 respectively.

The data of the problem, *p* and *s*, for *A* and *B* corresponding to the dates of observations of *C*, as shown in Table I, were read off from the position angle and distance curves as shown in the accompanying cut.

In constructing these curves I have used time as argument and the position angles and distances respectively as ordinates. The



data which I used are indicated in connection with the construction. In the distance curve there is a suggestion of an undulation, covering a period of about 25 years.

The observations of *A*, *B* and *A*, *C* have been reduced to the equinox of 1850.

TABLE I.

Adopted Coördinates *A*, *C*.

Date.	Nights.	θ ° ′	ρ ″	Weight.	Observer.	Reference.
1837.09	5	89 24.0	43.95	1	Σ	*b*2
40.58	2	89 20.5	45.22	1	"	"
51.55	11	88 58.3	47.88	1	0Σ	"
55.52	7	88 44.6	49.06	1	"	"
60.26	7	88 26.7	50.28	1	"	"
64.31	4	88 14.5	51.52	1	Δ	*i*
66.91	4	88 12.0	52.32	1	0Σ	*b*2
75.04	2	87 49.0	54.84	1	Δ	*i*
75.64	5	87 36.4	54.56	1	0Σ	*b*2
82.51	3	87 16.0	56.22	1	"	"
1902.63	3	86 14.0	61.85	1	C	*o*

From the above data observation equations were formed for

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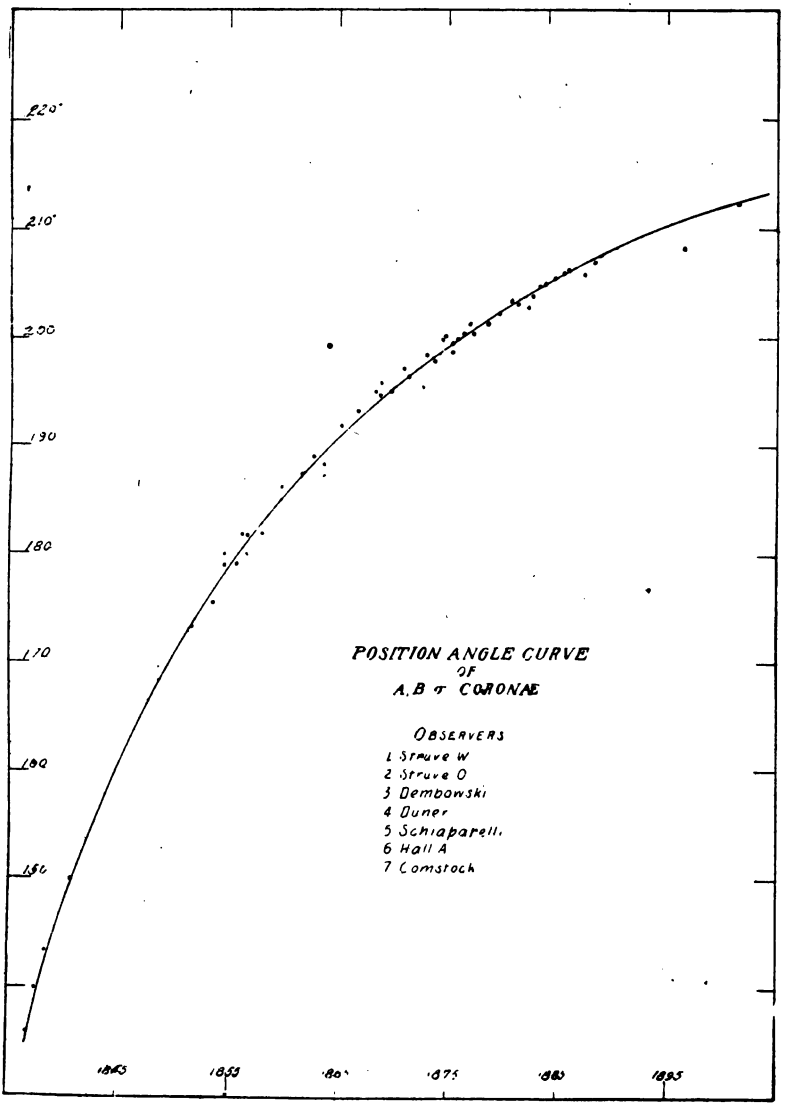
determining the five unknowns in equations (3). These treated



by the method of least squares furnish the following elimination equations:

In Declination.	In Right Ascension.
$x + 0.000\,y - 0.713\,z - 0.024 = 0$	$x' + 0.000\,y' - 0.365\,z' - 0.161 = 0$
$y - 0.430\,z - 0.054 = 0$	$y' - 0.411\,z' + 0.007 = 0$
$z - 0.007 = 0$	$z' - 0.106 = 0$

in which

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$x = \Delta\alpha$, $y = 37.90\,\Delta\beta$, $z = 3.64\,\Delta k$
$x' = \Delta\mu$, $y' = 37.90\,\Delta\nu$, $z' = 2.45\,\Delta k$.

Their solution yields

$\Delta\alpha = -0.018$, $\Delta\beta = +0.0015$, $\Delta k = +0.002$
$\Delta\mu = +0.200$, $\Delta\nu = +0.0009$, $\Delta k = +0.043$,

The assumed values were

$\alpha_0 = +3.055$, $\beta_0 = +0.0745$, $k_0 = +0.50$
$\mu_0 = +51.677$, $\nu_0 = +0.2973$,

Therefore,

$\alpha = +3''.037 \pm 0''.3078$ $\mu = +51''.877 \pm 0''.155$
$\beta = +0''.0760 \pm 0''.0100$ $\nu = +0''.2983 \pm 0''.0347$
$k = +0.502 \pm 0.064$ $k = +0.543 \pm 0.068$

I find for the probable error of an equation of unit weight

In Declination $r_1 = \pm 0.10$. In Right Ascension $r_1 = \pm 0.23$.

The weights of the various quantities are

$P_x = 0.105$, $P_y = 0.262$, $P_z = 0.054$, $P_{x'} = 2.217$, $P_{y'} = 1.170$, $P_{z'} = 0.369$
$P_\alpha = 0.105$, $P_\beta = 9.937$, $P_k = 0.197$, $P_\mu = 2.217$, $P_\nu = 44.340$, $P_k = 0.904$

I then found the probable errors of α, β, μ, ν and k, as given above. The same arrangement is followed in the case of the other stars treated in this paper.

As a result of this investigation I find for the mean value of k,

$$k = +0.52 \pm 0.065$$

which is accepted as the best the data will yield. This value is physically possible since from the nature of a center of gravity it must lie between the two bodies, from which it follows that k must lie between 0 and + 1 in order that it may have a physical interpretation. The result from the solution of such a set of equations is not subject to such limitations but might be equally well either positive or negative.

The time unit employed is one year and β and ν therefore represent annual variations. The values of α and μ given above are for the epoch 1864.73.

I have formed the sums of the squares of the residuals and found the probable error of an equation of unit weight on the hypothesis that $k = 0$ and $k = +0.52$ as follows:

In Declination.

For $k = 0$ $[pvv] = 13.08$ $r_1 = \pm 1.28$
$k = +0.52$ $[pvv] = 0.10$ $r_1 = \pm 0.11$

In Right Ascension.

For $k = 0$ $[pvv] = 1.01$ $r_1 = \pm 0.36$
$k = 0.52$ $[pvv] = 0.43$ $r_1 = \pm 0.24$

These results show a decided improvement due to assuming an orbital motion for A. Using the mean value of k, I find for the ratio of the masses of the components of σ Coronæ

$A : B$ 1 : 1.08

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III.

Determination of the Relative Masses in the System β Delphini.

$\alpha = 20^h\ 31^m.9 \qquad \beta = +14^\circ\ 11'$

This close pair was discovered by Burnham in 1883. Since that time the components of the binary have described more than a revolution around their common center of gravity, and although their maximum distance is less than two-thirds of a second of arc and their minimum less than one-fourth of a second, the star has been well observed during the last twenty years.

The magnitudes of A and B are 4.3 and 5.9 respectively. There is a neighboring star C of the 12.7 magnitude which was observed by Sir John Herschel in 1828. It was measured by Burnham in 1878 and seven other observations of it are recorded up to 1898. These observations are shown in Table II. The corrections have been applied for reducing the distances, ρ, and the position angles, θ, from $\frac{AB}{2}$ to A as origin and for the reduction of the position angles, θ, to the equinox of 1900. The data, p and s, of B with respect to A were derived for the dates of the available observations of C from Burnham's* orbit and period of 26.7 years.

Table II.

Adopted Coördinates of A, C.

Date.	Nights.	θ ° ′	ρ ″	Weights.	Observer.	Reference.
1878.05	3	115 51	27.71	1	β	1
81.19	4	115 42	27.68	1	"	"
83.54	1	116 31	27.50	.5	"	"
89.82	2	115 37	26.61	1	"	"
90.46	3	117 6	26.71	1	"	"
95.81	1	117 15	25.96	.5	L	"
98.49	3	117 12	25.63	1	β	"
98.59	2	116 47	25.68	1	Ba	"

From the data thus obtained observation equations were formed for determining the five unknowns in equations (3). These treated by the method of least squares yielded the following elimination equations.

In Declination.

$$x - 0.004\,y + 0.286\,z - 0.012 = 0$$
$$y + 0.380\,z + 0.143 = 0$$
$$z + 0.387 = 0$$

In Right Ascension.

$$x' - 0.004\,y - 0.020\,z - 0.117 = 0$$
$$y - 0.121\,z + 0.139 = 0$$
$$z + 0.681 = 0$$

in which

$$x = \Delta\alpha, \qquad y = 11.32\,\Delta\beta, \qquad z = \Delta k,$$
$$x' = \Delta\mu, \qquad y' = 11.32\,\Delta\nu, \qquad z' = \Delta k,$$

Their solution yields

$$\Delta\alpha = +0.123, \qquad \Delta\beta = +0\ 00035, \qquad \Delta k = -0.387,$$
$$\Delta\mu = -0.028, \qquad \Delta\nu = -0.01952, \qquad \Delta k = -0.681,$$

The assumed values were

$$\alpha_0 = -12.036, \qquad \beta_0 = +0.1582, \qquad k_0 = +0.50$$
$$\mu_0 = +23.776, \qquad \nu_0 = -0.09226,$$

* *Pub. Yerkes Observatory* Vol. I.

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Therefore,

$\alpha = -11''.913 \pm 0''.181$ $\mu = +23''.748 \pm 0''.063$
$\beta = +0''.01582 \pm 0''.00878$ $\nu = -0''.11178 \pm 0''.0255$
$k = +0.113 \pm 1.04$ $k = -0.181 \pm 2.65$

I find for the probable error of an equation of unit weight

In Declination $r_1 = \pm 0.26$ In Right Ascension $r_1 = \pm 0.21$

The weights of the various quantities are

$P_x = 2.06$, $P_y = 2.69$, $P_z = 0.274$, $P_{x'} = 6.983$, $P_{y'} = 2.446$, $P_{z'} = 0.163$,
$P_\alpha = 2.06$, $P_\beta = 28.80$, $P_k = 0.274$, $P_\mu = 6.983$, $P_\nu = 27.69$, $P_k = 0.163$,

I then found the probable errors of α, β, μ, ν and k as given above.

The value of k derived from the right ascension equations is physically impossible, and the large probable errors show that both values of k are wholly unreliable.

Another companion, D, of the eleventh magnitude was observed by William Struve in 1829. This has been fairly well observed in subsequent years and in this investigation eighteen observations of $\frac{AB}{2}$, D, as shown in the following table have been used. The data, p and s, of B with respect to A were derived for the dates of the available observations of D, from Burnham's orbit as referred to above.

Corrections have been applied for reducing the distances, ρ, and position angles, θ, from $\frac{AB}{2}$ to A as origin and for the reduction of the position angles, θ, to the equinox of 1900.

TABLE III.

Adopted Coördinates A, D.

Date.	Nights.	θ °	′	ρ ″	Weights.	Observer.	Reference.
1829.40	3	343	27	32.32	1	Σ	1
51.84	2	339	16	33.80	1	0Σ	"
64 61	2	336	25	34.83	1	Δ	*i*
65.60	1	336	38	34.85	1	"	"
73.75	1	336	7	35.29	1	"	"
76.85	1	334	48	35.48	1	0Σ	*b*2
77.66	2	335	22	36.29	1	Sp	*n*
77.77	1	336	00	35.63	1	Δ	*i*
80.49	5	334	46	35.34	1	β	*f*
83.54	1	334	3	35.37	1	"	"
88.82	2	333	3	36.50	1	"	1
95.81	2	331	15	36.95	1	L	*j*
96.63	1	333	8	37.52	1	Pr	*q*
98.49	3	331	37	37.46	1	β	1
98.71	1	332	32	37.26	1	Ba	"
98.75	1	333	32	37.60	1	L	*j*
1901.83	1	330	37	37.90	1	"	"
02.81	3	331	26	36.98	1	C	*o*

From the data thus obtained I formed observation equations for determining the five unknowns in equations (3).

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These treated by the method of least squares yielded the following elimination equations:

In Declination.	In Right Ascension.
$x + 0.000\,y + 0.383\,z - 0.378 = 0$	$x' + 0.000\,y' + 0.047\,z' + 0.425 = 0$
$y + 0.492\,z - 0.225 = 0$	$y' - 0.030\,z' + 0.575 = 0$
$z + 0.307 = 0$	$z' + 0.430 = 0$

in which

$$x = \Delta\alpha, \quad y = 51.898\,\Delta\beta, \quad z = \Delta k,$$
$$x' = \Delta\mu, \quad y' = 51.898\,\Delta\nu, \quad z' = \Delta k,$$

Their solution yields

$$\Delta\alpha = +0.490, \quad \Delta\beta = +0.0072, \quad \Delta k = -0.307$$
$$\Delta\mu = -0.4049, \quad \Delta\nu = -0.0113, \quad \Delta k = -0.430$$

The assumed values were

$$\alpha_0 = +14.782, \quad \beta_0 = +0.1047, \quad k_0 = +0.50,$$
$$\mu_0 = -32.115, \quad \nu_0 = -0.0214$$

Therefore,

$$\alpha = +15''.272 \pm 0''.193 \qquad \mu = -32''.520 \pm 0''.161$$
$$\beta = +0''.1119 \pm 0''.0508 \qquad \nu = -0''.0327 \pm 0''.0447$$
$$k = +0.193 \pm 0.884 \qquad k = +0.070 \pm 1.544$$

I find for the probable error of an equation of unit weight

In Declination $r_1 = \pm 0.46$. In Right Ascension $r_1 = \pm 0.50$.

The weights of the various quantities are as follows:

$P_x = 5.58$, $P_y = 1.56$, $P_z = 1.07$, $P_{x'} = 16.54$, $P_{y'} = 2.40$, $P_{z'} = 0.44$
$P_\alpha = 5.58$, $P_\beta = 80.83$, $P_k = 1.07$, $P_\mu = 16.54$, $P_\nu = 124.33$, $P_{k'} = 0.44$

The large probable errors in the values of k show that these values are wholly unreliable, on the other hand the values obtained for α, β, μ and ν are valuable results. The values of α and μ are for the epoch 1881.30.

IV.

Determination of the Relative Masses in the System δ Equulei.

$\alpha = 21^h\ 9.6^m \qquad \delta = +9^\circ\ 37'$

This is a close binary having a period of 5.7 years according to Hussey.* It was first observed by Otto Struve in 1852. The plane of the orbit is highly inclined to the line of sight and the orbit is so placed that the change in right ascension due to the orbital motion does not exceed 0″.25. The components *A* and *B* are of about the same magnitude, 4.5 and 5 respectively. The neighboring star *C* is of the 9.2 magnitude. The position of *C* relative to $\frac{AB}{2}$ was first measured by William Struve in 1829, and it has been pretty well observed at subsequent dates as shown in Table IV. Between the dates 1869.67 and 1880.60 an extensive gap occurs, covering a period of almost two revolu-

* Pub. *A. S. P.* Vol. XII.

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tions of the binary, in which no observations of C were found. In determining the five unknowns of equations (3) for this star the following observations have been used, in which the corrections have been applied for reducing the distances, ρ, and the position angles, θ, from $\frac{AB}{2}$ to A as origin, and for the reduction of the position angles, θ, to the equinox of 1900.

TABLE IV.

Adopted Coördinates A, C.

Date.	Nights.	θ ° ′	ρ ″	Weight.	Observer.	Reference.
1828.80	2	41 1	26.80	1	Σ	*c*
32.10	4	39 38	27.48	1	"	"
34.90	2	37 53	27.73	1	"	"
35.64	4	37 43	27.74	1	"	"
36.65	3	37 17	28.22	1	"	*f*
37.77	2	36 39	28.28	1	"	*c*
41.65	1	34 43	29.00	1	0Σ	*bl*
47.82	1	32 8	30.64	1	"	"
51.84	1	30 54	31.22	1	"	"
52.64	1	30 52	31.54	1	"	"
53.91	1	29 13	31.68	1	"	"
54.69	1	29 57	31.65	1	"	"
56.58	1	29 27	32.44	1	"	"
57.67	1	28 45	32.76	1	"	"
58.59	1	28 23	33.03	1	"	"
59.65	1	28 9	33.06	1	"	"
62.71	4	27 15	33.67	1	△	*i*
63.64	4	26 52	34.08	1	"	"
64.90	5	26 54	34.47	1	"	"
65.91	1	26 6	34.79	1	0Σ	*bl*
66.52	2	25 37	34.72	1	W	*a*
69.67	2	25 27	35.99	1	Du	*g*
80.60	5	22 41	38.15	1	β	*f*
81.46	3	22 17	38.95	1	"	"
83.65	3	22 2	39.26	1	Pe	*m* 107
87.71	2	21 9	40.28	1	HΣ	*d*
93.68	2	19 0	41.74	1	Gl	*e*
98.97	2	19 29	43.20	1	S	*m* 109
1902.71	3	27 35	43.86	1	C	*o*

The data, p and s, for the component B referred to A, were obtained from Hussey's* orbit for the dates of the available observations of C.

Observation equations were formed from these data, and solved by the method of least squares. This solution yielded the following elimination equations;

In Declination.	In Right Ascension.
$x + 0.000\,y + 0.256\,z + 0.014 = 0$	$x' + 0.000\,y' + 0.104\,z' - 0.004 = 0$
$y + 0.010\,z - 0.129 = 0$	$y' + 0.003\,z' - 0.340 = 0$
$z + 0.329 = 0$	$z' + 0.222 = 0$

in which

$$x = \Delta\alpha, \quad y = 41.916\,\Delta\beta, \quad z = \Delta k, \quad x' = \Delta\mu, \quad y' = 41.916\,\Delta\nu, \quad z' = \Delta k.$$

Their solution yields

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$\triangle\alpha = +0.070$, $\triangle\beta = +0.0032$ $\triangle k = -0.329$
$\triangle\mu = +0.027$, $\triangle\nu = +0.0081$ $\triangle k = -0.222$

The assumed values were

$\alpha_0 = +29.498$, $\beta_0 = +0.2917$, $k_0 = +0.50$
$\mu_0 = +15.548$, $\nu_0 = -0.0580$,

Therefore

$\alpha = +29''.568 \pm 0''.038$ $\mu = +15''.575 \pm 0''.029$
$\beta = +0''.2949 \pm 0''.0100$ $\nu = -0''.0499 \pm 0''.0161$
$k = +0.172 \pm 0.988$ $k = +0.279 \pm 1.378$

I find for the probable error of an equation of unit weight

In Declination $r_1 = \pm 0.165$. In Right Ascension $r_1 = \pm 0.267$.

The weights of the various quantities are as follows:

$P_x = 4.35$, $P_y = 6.57$, $P_z = 0.334$, $P_{x'} = 9.32$, $P_{y'} = 6.57$, $P_{z'} = 0.150$,
$P_\alpha = 4.35$, $P_\beta = 275.22$, $P_k = 0.334$, $P_\mu = 9.32$, $P_\nu = 275.22$, $P_k = 0.150$,

I then found the probable errors of α, β, μ, ν, and k as given above. The large probable errors in the values of k show that the data now available are wholly insufficient for determining the relative masses of the components of the binary δ Equulei. On the other hand the probable errors in the values of α, β, μ and ν indicate that these results are reliable. The values of α and μ given above are for the epoch 1860.79.

V.

Determination of the Relative Masses in the System τ Cygni.

$\alpha = 21^h\ 10^m$ $\delta = +37°\ 32''$

This was observed as a double star by Otto Struve in 1851. Several years later Alvan G. Clark discovered that the brighter of the two stars was a binary. It is known as A. G. C. 13. The magnitudes of the two components and of the neighboring star are respectively 4.2, 8.0 and 8.9. The binary has been well observed since the date of its discovery and the orbit seems to be well determined. The computed periods vary from 36.5 years to 57.25 years. I have obtained the data of the problem for *A*, *B* from Aitken's* orbit for the dates of the available observations of *C* and have used his period of 45.1 years in determining the position of *B* with respect to *A* at these dates. According to Aitken the length of the axis major of the apparent orbit is 1''.96 and of the axis minor 1''.20.

I have found only seven observations of *A*, *C*, six by Otto Struve between 1851 and 1883 and an unpublished one by Professor Comstock in 1903, as shown in the table below. I find no observation of *A*, *C*, when the position angle of *B* was between 0° and 90°, one when it was between 90° and 180°, two between

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180° and 270° and four between 270° and 360°. This gives but one positive coördinate in right ascension and six negative, while in declination there are four positive coördinates and three negative.

TABLE V.

Adopted Coördinates *A*, *C*.

Date.	Nights.	θ °	′	ρ ″	Weight.	Observer.	Reference.
1851.84	1	213	1	136.53	1	OΣ	*b*2
52.66	1	212	42	135.88	1	"	"
52.69	1	212	47	136.85	1	"	"
54.78	1	212	32	138.21	1	"	"
66.72	1	212	00	142.98	1	"	"
83.87	1	211	19	151.15	1	"	"
1903.52	3	210	56	160.15	1	C	*o*

The data thus obtained have been used for obtaining the five unknowns in equation (3), the position angles, θ, are reduced to the equinox of 1900. The solution of the observation equations thus formed furnish the following elimination equations:

In Declination.

$$x + 0.000\,y + 0.109\,z - 0.011 = 0$$
$$y - 0.590\,z + 0.010 = 0$$
$$z + 0.332 = 0$$

In Right Ascension.

$$x' + 0.000\,y' - 0.346\,z' - 0.441 = 0$$
$$y' + 0.227\,z' + 0.401 = 0$$
$$z' + 0.069 = 0$$

in which

$$x = \triangle\alpha, \qquad y = 36.94\,\triangle\beta, \qquad z = \triangle k$$
$$x' = \triangle\mu', \qquad y' = 36.94\,\triangle\nu, \qquad z' = \triangle k$$

Their solution yields

$$\triangle\alpha = +0.048, \quad \triangle\beta = -0.0053, \quad \triangle k = -0.332,$$
$$\triangle\mu = +0.417, \quad \triangle\nu = -0.0104, \quad \triangle k = -0.070,$$

The assumed values were

$$\alpha_0 = -121.219, \quad \beta_0 = -0.4361, \quad k_0 = +0.50$$
$$\mu_0 = -76.397, \quad \nu_0 = -0.1514,$$

Therefore

$$\alpha = -121''.171 \pm 0''.159 \qquad \mu = -75''.980 \pm 0''.231$$
$$\beta = -0''.4414 \pm 0''.0664 \qquad \nu = -0''.1618 \pm 0''.0574$$
$$k = +0.168 \pm 0.359 \qquad k = +0.430 \pm 0.450$$

I find for the probable error of a unit weight

In Declination $r_1 = \pm 0.41$. In Right Asceesion $r_1 = \pm 0.46$

The weights of the various quantities are as follows:

$P_x = 6.637$, $P_y = 1.035$, $P_z = 1.362$, $P_{x'} = 3.982$, $P_{y'} = 1.747$, $P_{z'} = 1.093$,
$P_\alpha = 6.637$, $P_\beta = 38.233$, $P_k = 1.362$, $P_\mu = 3.982$, $P_\nu = 64.568$, $P_k = 1.093$.

The large probable errors in the values of k show that the data at present available are insufficient for determining the relative masses of A and B τ Cygni. More extensive and well distributed observations will probably furnish sufficient data for the solution of this problem.

The values of α and μ found above are for the epoch 1866.58.

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VI.

Determination of the Relative Masses in the System κ Pegasi.

$\alpha = 21^h\ 39^m \qquad \delta = +25°\ 6'$

The close pair was detected by Burnham in 1880. Various computors have found periods between 11 and 12 years. The component B has described about two revolutions since its discovery and has been well observed during that time. It is always a close pair, the maximum distance being less than 0″.30. A and B are of about the same magnitude 4.5 and 5 respectively while C is of about the tenth magnitude. Sir William Herschel discovered the wide pair but it was first measured by William Struve in 1831. The observation of $\frac{AB}{2}$ and C used in this discussion are set forth in Table VI. The coördinates, p and s, for the component B with respect to A, for the dates corresponding to the observation of C, were derived from Burnham's* orbit. The position angles, θ, have been reduced t › the equinox of 1900.

TABLE VI.

Observed Coördinates *A*, *C*.

Date.	Nights.	θ ° ′	ρ ″	Weight.	Observer.	Reference.
1831.56	5	307 57	11.07	1	Σ	1
64.87	6	303 37	11.66	1	△	"
80.60	4	302 58	11.66	1	β	"
86.75	2	300 45	11.88	1	HΣ	*d*
88.81	3	299 44	12.18	1	Hl	1
88.82	2	300 26	12.27	1	β	"
88.92	2	300 32	12.00	1	Sp	"
93.66	2	297 29	11.71	1	Gl	"
94.82	2	298 50	12.05	1	L	*j*56
95.60	8	301 14	12.18	1	Ba	1
95.69	2	299 01	12.16	1	L	*j*56
96.70	1	297 48	12.13	1	"	*j*59
97.86	1	297 42	12.07	1	"	"
98.43	1	299 24	12.54	1	A	*n*150
98.71	3	299 38	12.56	1	L	*j*59
99.79	3	297 53	12.52	1	"	*j*60
1903.54	3	298 41	12.58	1	C	*o*

Observation equations were formed from these data and solved by the method of least squares. The solution yields the following elimination equations:

In Declination.	In Right Ascension.
$x + 0.000\,y - 0.025\,z + 0.161 = 0$	$x' + 0.000\,y' - 0.023\,z' - 0.179 = 0$
$y - 0.019\,z + 0.227 = 0$	$y' + 0.185\,z' - 0.012 = 0$
$z + 0.250 = 0$	$z' - 0.169 = 0$

in which

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$$x = \Delta\alpha, \quad y = 56.977\,\Delta\beta, \quad z = \Delta k,$$
$$x' = \Delta\mu, \quad y' = 56.977\,\Delta\nu, \quad z' = \Delta k.$$

Their solution yields

$$\Delta\alpha = -\ 0.167, \quad \Delta\beta = -\ 0.0040, \quad \Delta k = -\ 0.250,$$
$$\Delta\mu = +\ 0.183, \quad \Delta\nu = -\ 0.0054, \quad \Delta k = +\ 0.169.$$

The assumed values were

$$\alpha_0 = +\ 6.205, \quad \beta_0 = -\ 0.0107, \quad k_0 = +\ 0.50$$
$$\mu_0 = -\ 10.609, \quad \nu_0 = -\ 0.0345,$$

Therefore,

$$\alpha = +\ 6''.038 \pm 0''.187 \qquad \mu = -\ 10''.426 \pm 0''.120$$
$$\beta = -\ 0''.0147 \pm 0''.0273 \qquad \nu = -\ 0''.0399 \pm 0''.0178$$
$$k = +\ 0\ .250 \pm 3\ .174 \qquad k = +\ 0\ .669 \pm 0\ .774$$

I find for the probable error of an equation of unit weight

In Declination $r_1 = \pm\ 0.25$. In Right Ascension $r_1 = \pm\ 0.16$

The weights of the various quantities are as follows:

$P_x = 1.680$, $P_y = 1.456$, $P_z = 0.084$, $P_{x'} = 1.697$, $P_{y'} = 1.345$, $P_{z'} = 0.561$,
$P_\alpha = 1.680$, $P_\beta = 82.943$, $P_k = 0.084$, $P_\mu = 1.697$, $P_\nu = 76.620$, $P_k = 0.561$.

The above probable errors in the values of α, β, μ, ν and k were then computed. The large probable errors in the values of k indicate that the data now available are wholly insufficient for the determination of the relative masses of components of κ Pegasi. The values of α and μ found above are for the epoch 1888. 54.

I have examined all the available records of micrometrical measurements of double star observations accessible to me and the above summary contains all the stars for which sufficient data were found to justify an attempt to determine the ratio of the masses of the components of the binary by the method here employed. I have also examined the records with a view of finding other binaries having optical companions for which additional measures of the binary and companion would probably furnish data for determining the relative masses of the components in the comparatively near future; but the search failed to bring any such to light.

The most pressing cases seemed to be the following for which additional observations are desired. The first column contains the dates of the measures given.

				θ	ρ	Mag.	Remarks.
1902.81	β Delphini	$\frac{AB}{2}$, D	331°	7′	36″.86	11	Once every one or two years.
1898.59	"	$\frac{AB}{2}$, C	117	6	25 .92	12	
1902.71	δ Equulei	$\frac{AB}{2}$, C	17	30	43 .74	9.2	Once each year or oftener.
1903.52	τ Cygni	A,C	210	55	160 .15	89.	Once in two or three years.

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VII.

SUMMARY OF RESULTS.

Star.	R.A.	Decl.	Magnitudes.			R.A. Coördinates.		Decl. Coördinates.	
	h m	° ′				k	Weight.	k	Weight.
σ Coronae	16 11	+34 7	4.5	5.0	10.2	+0.543 ± 0.064	0.90	+0.502 ± 0.068	0.20
β Delphini (A,B,C)	20 31.9	+14 11	4.3	5.9	12.7	−0.181 ± 2.65	0.16	+0.113 ± 1.04	0.27
" (A,B,D)			4.3	5.9	11.0	+0.070 ± 1.54	0.44	+0.193 ± 0.88	1.07
δ Equulei	21 9.6	+ 9 36	4.5	5.0	9.2	+0.279 ± 1.38	0.15	+0.117 ± 0.998	0.33
τ Cygni	21 10	+37 32	4.2	8.0	8.9	+0.430 ± 0.450	1.09	+0.168 ± 0.359	1.36
κ Pegasi	21 39	+25 6	4.5	5.0	10.0	+0.699 ± 0.774	0.56	+0.250 ± 3.174	0.08

Star.	Epoch.	Motion in R.A.				Motion in Decl.			
		μ	"	ν	"	α	"	β	"
σ Coronae	1864.73	+51.877	± 0.155	+0.2983	± 0.0347	+ 3.037	± 0.308	+0.0760	± 0.0100
β Delphini (A,B,C)	1889.37	+23.748	± 0.063	−0.1118	± 0.0255	− 11.913	± 0.181	+0.0158	± 0.0088
" (A,B,D)	1881.30	−32.512	± 0.161	−0.0327	± 0.0447	+ 15.272	± 0.193	+0.1119	± 0.0508
δ Equulei	1860.79	+15.575	± 0.029	−0.0499	± 0.0161	+ 29.568	± 0.038	+0.2949	± 0.0100
τ Cygni	1866.58	−75.980	± 0.231	−0.1618	± 0.0574	−121.171	± 0.159	−0.4414	± 0.0064
κ Pegasi	1888.54	−10.426	± 0.120	−0.0399	± 0.0178	+ 6.038	± 0.187	−0.0147	± 0.0273

VIII.

DETERMINATION OF MASS FROM OBSERVATIONS OF ABSOLUTE POSITIONS.

In a few cases it has been found feasible to determine from meridian circle observations the apparent orbit of one component of a binary star. When this has been done a very few micro-

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metrical measurements of the two components of the binary will suffice to determine their relative masses. For the sake of completeness there is therefore appended an outline of the determination of such orbits from meridian circle observations following in substance the method of Peters and Auwers as set forth in the *Astronomische Nachrichten* Nos. 746, 747 and 1372.

Let the plane through the center of gravity, O, of the binary system and perpendicular to the line of sight be the plane xy, the positive direction of the x-axis being directed toward the north and of the y-axis toward the east.

Let δ = the declination of the star at the time t,
D = the declination of O at the time t,
α = the R. A. of the star at the time t,
A = the R. A. of O at the time t,

Put

$$\delta - D = \xi \text{ and } 15\,(\alpha - A) = \eta$$

then for the time t,

$$x = \xi \text{ and } y = \eta \cos \delta.$$

x and y are functions of the elements of the orbit, if u is the eccentric anomaly then

$$\begin{aligned} y = \eta \cos \delta &= (f + g \cos u + h \sin u) \cos \delta \\ x = \xi \qquad &= f' + g' \cos u + h' \sin u \end{aligned}$$

where according to Peters *A. N.* No. 746

$$\begin{aligned} g &= a \sec \delta\,(\cos w \sin \Omega + \sin w \cos \Omega \cos i) \\ h &= -a \sec \delta \sqrt{1-e^2}\,(\sin w \sin \Omega - \cos w \cos \Omega \cos i) \\ f &= -ge \\ g' &= a\,(\cos w \cos \Omega - \sin w \sin \Omega \cos i) \\ h' &= -a \sqrt{1-e^2}\,(\sin w \cos \Omega + \cos w \sin \Omega \cos i) \\ f' &= -g'e \end{aligned}$$

Let δ^0 = the mean declination of the star at the time t^0,
δ_r^0 = the mean declination of the star at the time t^0 computed from reduction tables.

Put

$$\delta^0 - \delta_r^0 = k'$$

Then the declination of the center of gravity, O, at the time t^0 equals $\delta_r^0 + k' - \xi^0$, or

$$D^0 = \delta_r^0 + k' - (f' + g' \cos u + h' \sin u)$$

Let c' = the annual change in the declination of O derived from the annual change in the declination of the star,
δ_r = the declination of the star at the time t computed from reduction tables.

Then for the time t

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$$D = \delta_r + k' + c'(t - t^0) - (f' + g' \cos u + h' \sin u)$$

Now

$$\delta = D + \xi$$

Hence

$$\delta = \delta_r + k' + c'(t - t^0) + g'(\cos u - \cos u_0) + h'(\sin u - \sin u_0)$$

Each observation will make known $\delta - \delta_r$, and will furnish an observation equation of the above type.

If the R. A. has been observed the corresponding equation is

$$\alpha = \alpha_r + k + c(t - t^0) + \frac{1}{15} g(\cos u - \cos u^0) + \frac{1}{15} h(\sin u - \sin u^0)$$

where α_r is the R. A. of the star computed from reduction tables. If we place

$$k' - (g' \cos u^0 + h' \sin u^0) = b' + f'$$

$$k - \frac{1}{15}(g \cos u^0 + h \sin u^0) = b + f$$

$$\delta - \delta_r = r$$

$$\alpha - \alpha_r = q$$

the above equations take the form

$$r = b' + f' + c'(t - t^0) + g' \cos u + h' \sin u \qquad (1)$$

$$q = b + f + c(t - t^0) + \frac{1}{15} g \cos u + h \sin u \qquad (2)$$

If T is the epoch of peri-astron and n the mean yearly motion of the star in its orbit the mean anomaly, at the time t, equals $n(t - T)$ and is connected with the eccentric anomaly by the relation

$$n(t - T) = u - e \sin u \qquad (3)$$

r is a function of the unknowns n, T, e, $(b' + f')$, c', g' *and* h'; q is a function of the unknowns, n, T, e, $(b + f)$, c, g and h.

Each observation in declination will furnish an equation of condition of the form (1), and each observation in right ascension will furnish an equation of condition of the form (2).

A set of equations of condition formed from a sufficient number of observations in declination or in right ascension, with equation (3) will yield values of the unknowns and thus lead to the elements of the orbit.

After the orbit of one of the components has been found, a few micrometrical measurements of the two components will suffice to make known the relative masses by the following simple formula.

$$\frac{a'}{s} = \frac{M_1}{M_1 + M_2}$$

where M_1 and M_2 represent the masses of the two components of the binary, a^1 represents the radius vector of the known orbit at the time the observation is made and s the distance between the two components at that time.

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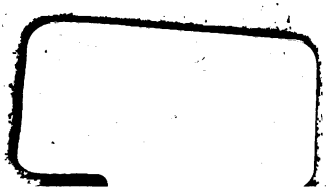
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